TEMPERATURE AND PRESSURE DISTRIBUTION ANALYSIS ON DEFORMABLE SHORT HYDRODYNAMIC BEARINGS

L.Carvalho Greco¹, R.Martins de Souza¹, R. Lima Stoeterau¹
¹Department of Mechanical Engineering, Polytechnic School of the University of São Paulo, São Paulo, Brazil

ABSTRACT
This paper presents a discussion on the study of thermal and elastic effects resulting from the support bearing. It was proposed a mathematical model based on the short bearings equations within the cavitation region, using the principle of mass continuity. Then, the used equations are deduced from [1] and energy equations applying a modified solution for the Ocvirk’s solution [2]. All the equations are numerically solved by the Backward Finite Difference Method. In addition of the treatment of fluid mechanics, this paper agrees the thermal analysis on bearings discussed by [3]. To calculate the deformation of the structure, it was used a Finite Element Model for a specific geometry and an evaluation of the variation of the pressure field is done to determinate how this difference affects other fluid properties. Finally, it was calculated within the full model to predict the short bearing properties. Comparing the similarities among the presented by [4] and the calculated ones, the mean results present a difference of 2.33% between the maximum pressure, and one about 1.78% for eccentricity ratio for a length-diameter bearing ratio (LDR) of ½. These results demonstrates that the model has an interesting relevant and accurate approximation for experimental data, reducing the time for calculate some initial properties or making qualitative decisions about its operation.

KEY WORDS: short bearing, modeling, pressure distribution.

DEFINITIONS
- Radial clearance [m]
- e Distance between the geometrical centers of shaft and bearing [m]
- h Film thickness [m]
- h_cor Film thickness correction factor [m]
- h_d Mechanical deformation [m]
- h_e Thermal deformation [m]
- p Bearing pressure [Pa]
- (x,y) Principal axis [m]
- α Thermal expansion factor [Pa⁻¹]
- ε = e/c Relative eccentricity
- δ Shaft deformation [m]
- η Fluid viscosity [Pa.s]
- θ Radial bearing co-ordinate [rad]
- θ_cor Correction co-ordinate [rad]
- ν Poison number
- φ Attitude angle [º]
- ω Angular speed [s⁻¹]
1.- INTRODUCTION
The works conducted until the middle of the twentieth century [5, 6, 7, 8, 9] provided an understanding of an important number of specific cases of the use of bearings and their approximations. However, since the second half of the twentieth century, the need to further explore the phenomenon of lubrication has become more important, due to technological breakthroughs in the area of transportation and the increasing changes in the demand for products. During this period, many works have been developed to understand the coupling of lubrication properties and their interaction with the structure and the energy equations [10, 11, 12] and various correction models were proposed [13, 14].
Currently, the majority of lubrication works is dedicated to the study of automotive engines [15, 16], focusing on a better understanding of fluid-structure interaction. If one analyzes the current trends of reduction in the size of engines and increase in the power supplied, the use of the theory of short bearings becomes increasingly interesting. This type of theory requires less processing capability, and provides good results, when compared with the theory of finite bearings for relations L/D smaller than 0.25. The present paper will explore the properties of short bearings, comparing a proposal method of design with experimental data provided by [4].
In this work, it’s used the equations proposed by [17] for the description of the velocity field, for heat dissipation in working fluid - as well as heat exchange with the other components of the structure - and the thermal deformation equations. For the determination of the pressure field, the basis equation of short bearing was taken and it was modified to couple the changes in the film thickness determination. In addition, density and viscosity correction models were used for a better interpretation of fluid properties. To calculate the pressure field is adopted a mobile coordinate system with zero value for the angular position in opposition to the pressure peak, so the angle of attitude does not influence the calculation of the properties.
Therefore, this paper proposes a model based on the modification of the equation of short bearings, incorporating a method of determining the viscosity correction for pressure and temperature, the correction of the temperature throughout the bearing profile and the deformations resulting from temperature and pressure. This modification of the classical solution is proposed as a way to expand the scope of the validity of the assumptions relating to short bearings and reduce errors in approximation by finite bearings in L/D relations above 0.25.
2.- THERMO-HYDRODYNAMIC EQUATIONS

2.1.- Initial Model
The classical Ocvirk’s solution was designed for bearings with a LDR not greater than $\frac{1}{4}$, nevertheless some authors say that this formulation could be used until the LDR of $\frac{1}{2}$ with an acceptable error. In order to reduce the error, the same calculus method proposed by Ocvirk’s hypothesis was adopted, wherein the pressure variation along the length is much larger than in the radial direction, resulting in equation (1). As a way to make more accurate equation, only two variables could be changed: the viscosity and film thickness. Therefore, there are proposed a viscosity correction model and an alternative model for the determination of film thickness.

$$p = \frac{3\eta U \frac{dh}{dx}}{h^3} \left(y^2 - \left(\frac{L}{2}\right)^2\right)$$

(1)

2.2.- Viscosity determination
To compare the model with experimental results, we carried out the equalization of lubricating viscosity variation curve. Based on figure 1, equation (2) was obtained for fluid viscosity versus temperature in $10^{-3}$ Pascal. Although figure 1 presents the data in Fahrenheit degrees, the equation (2) was converted to Kelvin.

$$\eta(T) = 0.0015T^2 - 1.0319T + 177.01$$

(2)

2.3.- Film thickness
The term traditionally used to describe the film thickness was modified to include a correction term and the mechanical and thermal deformations. Initially, based on figure 2 and assuming that the shaft and bearing are dimensionally stable, the value of the film thickness ($h$) can be obtained for any position by applying geometry laws.

$$h(\theta, \text{def}, T) = h(\theta) + h_{\text{cor}}(\theta) + h_{\text{d}}(\theta) + h_{\text{t}}(\theta)$$

(3)

Figure 1. Diesel fuel viscosity chart. Extracted from [4].
In equation (3) the first term represents the term traditionally used for film thickness and the second is a traditionally despised one, because its order is smaller than the first one. Comparing the error committed by despising the second term it could be seen in the figure 3, calculated to the relation $\epsilon \sin(\theta) = 1$. Although the maximum error is still very small, by introducing this error in the classic solution for short bearings (equation 1) and comparing the values, it could be seen an error about 1.5% in the prediction of the maximum pressure. Although the error remains small, this paper aims to approach the most of the experimental results; the term referring to the error will be incorporated into the model as a means of correction for the pressure field values.

$$h(\theta) = c(1 + \epsilon \cos(\theta)) - R \left(1 - \sqrt{1 - \left(\epsilon \frac{c}{R} \sin(\theta)\right)^2}\right)$$  \hspace{1cm} (4)

In spite of the consideration of non-deformable structures, there are deformations in the shaft and bearing when the pressure field is formed. This well-known assumption is not valid when the pressure fields reach values close to 10 MPa. As each point of the pressure deformation depends mainly on the point itself, then, it can be inserted in the film thickness equation, represented by the third term of the equation (3) ($h_d(\theta)$), which will be calculated later.
Similarly to deformation, the temperature also causes a change in film thickness; it causes a material expansion and modifies the initial clearance between the pieces. The shaft and housing deformation will be represented by $h_t(\theta)$, which will be decomposed in each of the expansion terms.

2.4.- Mechanical deformation model

The mechanical deformation of the assembly can be envisaged as a bending phenomenon (shaft) and a deformation one (bearing housing). As shown in figure 4 (a), it is possible to separate the portion that interacts with the bearing from the shaft, modeling separately from the rest of the structure. One way to model this part of the shaft is to treat the shaft portion as a beam (figure 4 (b)). Therefore, it is possible to predict the deflection caused by the bearing load. Equation (5) is written based on the elastic beam theory for shaft deformation.

\begin{equation}
E\delta(y) = -\frac{\eta U}{\pi h^3} \left( \frac{D}{2} - c \right)^4 \frac{y}{30} \left( 3L^5 - 5L^3y^2 + 3L^4y - y^5 \right) \tag{5}
\end{equation}

In turn, for calculating the housing deformation, it was assumed a support structure representation for the bearing, as shown in figure (5). In order to avoid unnecessary deformation calculus of all structure points, it was used the Finite Element Model (FEM) to determine the stiffness matrix of the housing. After its determination, the resulting deformation was calculated for the applied loads which will be compared with theory and the values were inserted into the computer code.

2.5.- Thermal deformation model

According to [17], the only significant component of the thermal deformation is calculated in the radial direction of the structures, because the deformation in the longitudinal direction does not interact with film thickness. Calculating the heat exchange between the fluid and the surface, the equations (6) and (7) are established to the shaft and the bearing deformations, respectively.
\[ d(R_{\text{shaft}}) = \frac{2\alpha_{\text{shaft}}(1 + \nu)}{R_{\text{shaft}}} K_{h} \int_{0}^{R_{\text{shaft}}} \bar{r} \Delta T_{\text{shaft}}(\bar{r}) d\bar{r} \]  
\[ d(R_{\text{housing}}) = \frac{2\alpha_{\text{housing}}(1 + \nu)}{R_{\text{housing}}} K_{h} \int_{0}^{R_{\text{housing}}} \bar{r} \Delta T_{\text{housing}}(\bar{r}) d\bar{r} \]  

2.6 - Bearing-house Model

For the implementation of the thermal bearing equation, it is considered that the bearing housing and the bearing form a single piece with the radius equal to the distance between the inner surface and the outer side of the bearing housing. Using the structure of figure 5, it was modeled an equivalent radius to the equation (7). The equivalent radius has a condition, equation (8), which governs the system and allows the radius to be written according to Equation (9), otherwise the radius behaves according to equation (10). The housing was placed as a way to favor heat exchange with the environment, such that there is a faster balance in bearing temperature. The length of the housing was assumed equal to the bearing and the other dimensions were determined based on a squared housing, in order not to be too large.

\[
\cos(\theta_{\text{cor}}) \geq \frac{L_{2}}{\left(L_{2}^{2} + L_{2}^{2} + \left(\frac{D}{2}\right)^{2} + L_{1}D\right)^{0.5}}
\]

\[ R_{\text{housing}} = \frac{L_{2} - D\cos(\theta_{\text{cor}})}{\cos(\theta_{\text{cor}})} \]  
\[ R_{\text{housing}} = \frac{L_{1} + D(1 - \sin(\theta_{\text{cor}}))}{\sin(\theta_{\text{cor}})} \]  
\[ L_{2} = 1.5D; \ L_{1} = 1.5D \]

3.- RESULTS AND DISCUSSION

To evaluate the proposed model, the results were compared with experimental results ([4]), highlighting three experiments: the first experiment consists in a comparison with an experiment with large inlet pressure compared with the Ocvirk’s solution; the second is compared to a virtually pressurized bearing; finally, the third is compared with a bearing with an upper load and closer to the limit of validity of the traditional Ocvirk’s equation. In order to better visualize and compare the results, they were superimposed on the chart of [4]. In figures 6, 7 and 8, the dashed lines indicate the results for Ocvirk’s solution, the solid lines represent the experimental results and the colored lines represent the results obtained by the proposed model. Observing figures 6, 7 and 8 can be noted that experimental results not show the error of the Bourdon gauge used. In the same way, it is not clear if the pressure related come from a single measurement or the mean of measurements at the position, because it can be seen for some cases that there is more than one measurement in the same position. Also comparing the obtained curves and the experimental one in figures 6, 7 and 8 it can be seen that in all the lower pressure regions has a worse correlation between the shapes. This effect can be explained by the existence of the low accuracy to determine the fluid properties with regard to pressure and to an inaccuracy in the determination of the cavitation pressure.
Although it is possible to calculate the pressure field at any point of the bearing, when fluid cavitation occurs, the pressure becomes constant due to the presence of two-phase fluid. The experiments for comparing the Ocvirk’s model, shown by [4], showed no such behavior, however, the Bourdon gauge used by [4] cannot measure values less than 0.35 kPa (5 psia). Then, this limitation value was taken as the pressure at which the fluid undergoes cavitation. Table 1 summarizes the main properties studied.

Table 1. Summary of available variables.

<table>
<thead>
<tr>
<th>Data</th>
<th>Case 3.1</th>
<th>Case 3.2</th>
<th>Case 3.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auksmann</td>
<td>P (∫L/2) (psia)</td>
<td>e</td>
<td>P (∫L/2) (psia)</td>
</tr>
<tr>
<td>Proposed Model</td>
<td>45.00</td>
<td>0.3370</td>
<td>48.00</td>
</tr>
<tr>
<td>Error</td>
<td>2.33%</td>
<td>1.78%</td>
<td>0.10%</td>
</tr>
</tbody>
</table>

3.1.- Case 1
Analyzing the figure 6, it can be seen a better correlation between the experimental results and the results of the proposed model compared to Ocvirk’s solution ones. It is noted that the three obtained curves are almost entirely superimposed on the experimental curves and how bigger pressure level is, higher the correlation is. Calculating the error in peak pressure of each curve, it follows that with the pressure increasing the error is reduced, reaching become very close to zero at the maximum pressure point. Based on the curve ¾L, the maximum error is 6.25%, which is less than half the error reported by Ocvirk’s solution. In other hand, in the middle of the bearing the error reaches only 2.33% between the proposed model and the experimental data, although the Ocvirk’s solution has an error nearest to 10%.

Figure 6. Experimental pressure chart (case 3.1) compared to the proposed model results.
Comparing other variables of interest, it is noted that the eccentricity value reached a value of 0.343, which represents a 1.78% error. On the other hand, the calculated average viscosity has the value of 2.414 mPa while the value given by [4] is 4.81 mPa. Despite the difference, [4] states that the viscosity is calculated based on the temperature of the fluid container, which is clearly not the working temperature, so that any comparison with the viscosity or temperature becomes inaccurate.

### 3.2.- Case 2

Analyzing the Figure 6, there is a perceived very strong correlation between the experimental data and those obtained by the model. Moreover, both the peak pressure error (0.1%) and the eccentricity one (4.97%) were quite lower than the Ocvirk’s solution. The only difference between the prediction and the experimental results occurred in the lower pressure region where the curves showed less correlation.

It should be noted that the curves for the \( \frac{1}{4} \) \( L \) and \( \frac{3}{4} \) \( L \) for the forecast model are closer than the experimental ones. According to [4], the curves of \( \frac{1}{4} \) \( L \) and \( \frac{3}{4} \) \( L \) show differences because the fluid enters along the bearing, so the total pressure of the inlet side is greater than the output. Therefore, [4] calculates a linear pressure drop proportional to the distance from the leading edge, such that it covered half the system has a loss of 50% in the pressure difference between the inlet and the outlet edges. Therefore, by symmetry of the bearing, the difference between the two curves should have a fixed value of 0.5 psia, which does not occur. As shown by the experimental curves the difference varies considerably among the angular bearing positions, this variation is due to measurement error linked to the measuring equipment, making the curves keep way or get close more than the value predicted in theory.

Figure 7. Experimental pressure chart (case 3.2) compared to the proposed model results.
3.3.- Case 3
Analyzing the figure 8, it can be seen that although the maximum pressure error is only 0.39% and the eccentricity error is 3.29%, the shape of the curves is quite different and the predicted peak pressure occurs before the experiment peaks. Furthermore, it is possible to note that the experimental curves are not near to all measurements, unlike the previous ones, which could modify the experimental curve shape. This effect might be explained by the accuracy of the Bourdon gauge, which was subjected to maximum pressure almost two times higher than in the previous cases and minimum pressures lower than the measurement limit. Despite the lower correlation between the profiles, it can be seen a good correlation in the reconstitution of the fluid region and a greater predicted cavitation zone than the possibly one existing in the experiment.
This result shows that when the fluid is subjected to more drastic conditions, it is necessary a better knowledge of its properties. Therefore, the prediction model may prove more assertiveness in the pressure field determination and possibly on the size of the cavitation zone.

Figure 8. Experimental pressure chart (case 3.3) compared to the proposed model results.

4.- CONCLUSIONS
As shown, the proposed model was able to approach the experimental curves. It also presented a much smaller deviation than the values calculated by [4] for Ocvirk’s solution. It should be mentioned that one of the likely sources of deviations could be due to the lack of information on the error of the used instruments. This lack of information prevents the curves can be evaluated with the measurement error range, verifying if the proposed model is between the maximum and minimum deviation values of the measured pressure. Moreover, the lack of information concerning the temperature inside the bearing and the lubricant viscosity behavior hinder a better determination of viscosity and comparison of the thermal model.
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REFERENCES